Nonlinear Structural Response to Sonic Booms

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Introduction

A CONSIDERABLE amount of literature exists on the linear dynamic response of discrete and continuous systems subjected to sonic booms. The linear dynamic response of a spring-mass system subjected to sonic booms has been studied by Crocker and Hudson.[1-3] The problems of linear axisymmetric vibrations of thin circular elastic plates due to sonic boom excitation assuming small deflection theory has been analyzed by Pavagadhi and Yajnik[4] and Parnes.[5] The nonlinear analysis of rectangular panels subjected to sonic booms has been investigated by Alwar and Adi Murthy, using the timewise differentiation technique,[6,7] and the effects of overpressure, rise-time, and duration of sonic booms on the response have been studied. The results of the above analysis show that, for a practical case, the dynamic response due to sonic boom is in the nonlinear range.

In the present investigation, the influence of the large amplitude on the response of an isotropic circular plate subjected to sonic booms has been studied, including the influence of damping. It should be noted that the influence of damping on the response to sonic booms had been analyzed earlier[2] for a linear case treating the continuum as a spring-mass system. The influence of nonlinearity on the response has been studied and compared with linear response results taking into account the various boom parameters such as overpressure, duration, and rise time. Numerical results are presented for clamped edge boundary conditions in terms of the ratio of maximum linear to maximum nonlinear response.

Analysis

The governing differential equations for nonlinear transient response of a circular plate have been solved using the rapidly convergent Chebyshev series spacewise and the Houbolt[8] scheme in the timewise direction. The details of the application of the Chebyshev series for the static and dynamic response of circular plates have been given in Refs. 9 and 10.

Results and Discussion

In order to have a check on the present analysis, results obtained for the linear case are compared with the results given by Parnes[5] for a simply supported circular plate subjected to sonic boom, and there is good agreement between the two results. For the sake of brevity the results have not been presented here.

The large amplitude analysis is done for a practical case of a circular glass plate with the following data: $a/h=400$, $\nu=0.30$, $E=10^7$ psi, $\gamma=0.236\times10^{-3}$ lb-s^2/in.[4]

The effect of overpressure on the linear and nonlinear response of a clamped circular plate subjected to a symmetric *N* wave is plotted in Fig. 1. It can be seen that for an overpressure of 2 lb/ft^2 and positive pulse durations of 0.1 s, which are the values for a medium aircraft flying at normal cruising altitudes, the maximum dynamic deflection is of the order of the thickness of the plate and, hence, in the nonlinear range. The maximum stress at the center is of the order of 800 psi, which is less than the breaking strength of the glass.

In the present analysis, the influence of duration and rise time of the sonic boom pulse on the three phases—namely, positive forced maximum, positive free maximum, and

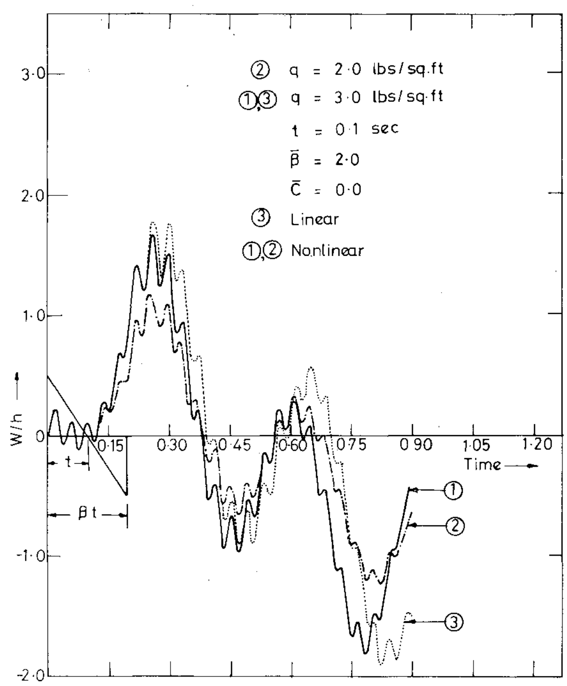

Fig. 1 Influence of overpressure to *N* wave pulse.

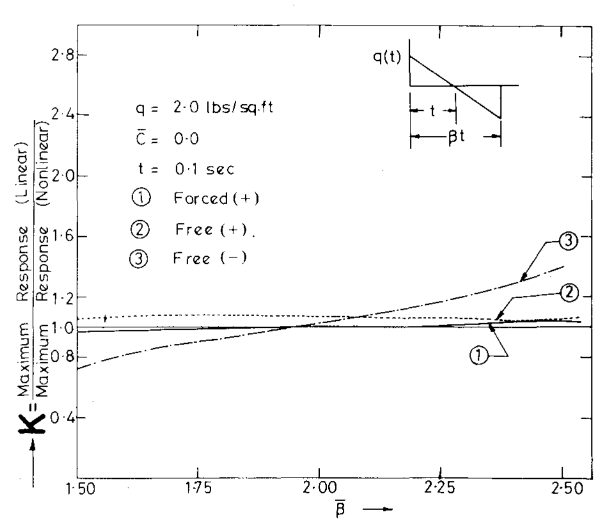

Fig. 2 Variation of *K* with respect to $\bar{\beta}$.

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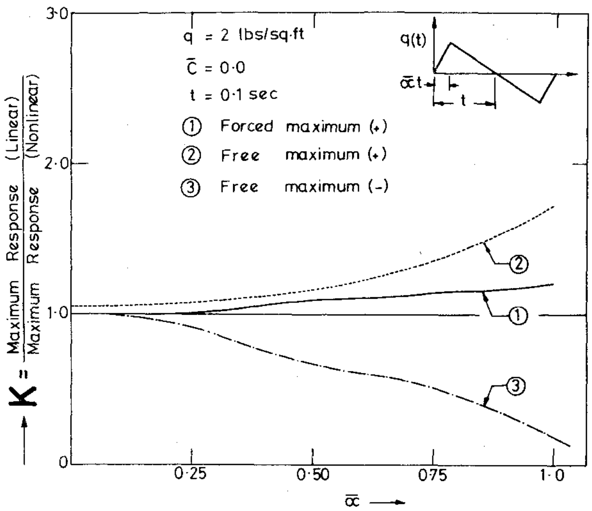

Fig. 3 Variation of K with respect to $\bar{\alpha}$.

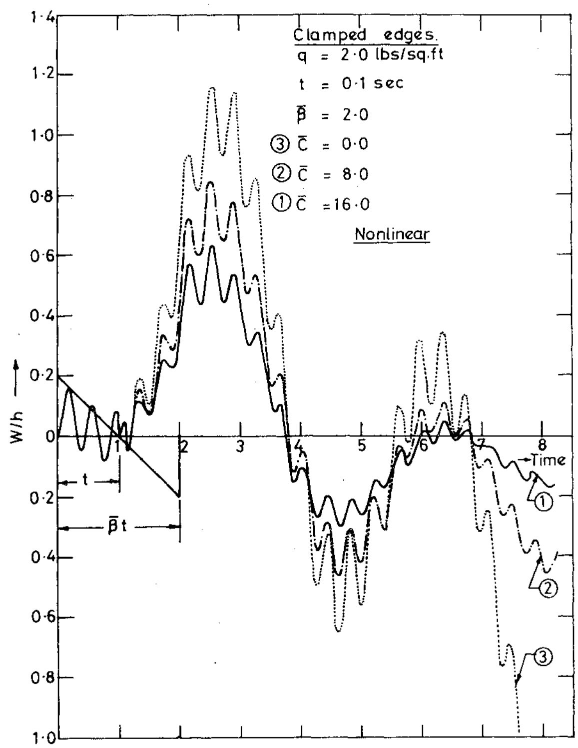

Fig. 4 Influence of damping.

negative free maximum—in terms of amplitude factor K has been studied. K is defined as the ratio of maximum linear to maximum nonlinear response. The results for a clamped circular plate are presented. Figure 2 shows the variation of K with duration parameters $\bar{\beta}$. It is seen that K is nearly constant with $\bar{\beta}$ for the first and second phases of motion, and it increases with $\bar{\beta}$ during the third phase of motion for the full range, up to $\bar{\beta}=2.5$.

The variation of K with the rise time parameter $\bar{\alpha}$ is shown in Fig. 3. In the first and second phases of motion, there is an increase in the magnitude of K with $\bar{\alpha}$, while in the thrid phase of motion K decreases.

Figure 4 shows the influence of damping on the response of a typical sonic boom pulse. The results are given for damping factors $\bar{C}$ equal to 0, 8, 16, the critical damping factor being 40. The influence of damping on the response is quite appreciable. Even a small value of $\bar{C}=8$ causes a reduction of about 23% in the magnitude as compared to undamped response, for an overpressure = 2 lb/ft^2 and positive duration of 0.1 s, which are the values given for a medium-size aircraft flying at normal cruising altitude. It can be seen that the influence of damping on the response is more pronounced in free motion than in forced motion.

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